



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

QUANTITATIVE INVESTMENT MANAGEMENT

LECTURE 59

Interest Rate Futures – 1

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INTEREST RATE FUTURES



DISCOUNT INSTRUMENTS

- There are two types of money market instruments:
- **Interest-bearing instruments**
- (e.g., bank certificates of deposit), and
- **Pure discount instruments**
- (e.g., Treasury bills).

YIELD ON AN INVESTMENT

- The yield on an investment is usually measured as the appreciation in the value of the investment
- **per unit of money**
- **per unit of time**
- Hence, we can say that the yield is the appreciation in value normalized with respect to money and time units.

MEASURES OF YIELD	S No	Measure	Money Normalization	Time Normalization
	1	Discount Yield	Final Value Exit Value	360 day year, Simple Interest
	2	Holding Period Yield	Initial Value Entry Value	No normalization
	3	Effective Annual Yield	Initial Value Entry Value	365 day year Compounded
	4	Money Market Yield	Initial Value Entry Value	360 day year Simple interest

$$\text{BDY}(r_{\text{BD}}) = \frac{\text{Discount}}{\text{Face Value}} \times \frac{360}{t} = \frac{F - P_0}{F} \times \frac{360}{t}$$

$$\text{HPY} = \frac{P_1 - P_0 + D_1}{P_0}$$

$$\text{EAY} = \left(1 + \text{HPY}\right)^{\frac{365}{t}} - 1$$

$$r_{\text{MM}} = \text{HPY} \times \frac{360}{t} = \frac{P_1 - P_0}{P_0} \times \frac{360}{t}$$

$$r_{MM} = \text{HPY} \times \frac{360}{t} = \frac{P_1 - P_0}{P_0} \times \frac{360}{t} = \frac{\left(\frac{P_1 - P_0}{P_1} \right)}{\left(\frac{P_0}{P_1} \right)} \times \frac{360}{t}$$

$$= \frac{\left(\frac{\text{BDY} \times t}{360} \right)}{\left(1 - \frac{\text{BDY} \times t}{360} \right)} \times \frac{360}{t} = \frac{\text{BDY} \times 360}{360 - \text{BDY} \times t}$$

By definition of BDY

$$\text{BDY}(\mathbf{d}) = \frac{\mathbf{P}_1 - \mathbf{P}_0}{\mathbf{P}_1} \times \frac{360}{\mathbf{N}_{\text{SM}}} \quad \text{so that} \quad \frac{\mathbf{P}_1}{\mathbf{P}_0} = \left(1 - \frac{\mathbf{dN}_{\text{SM}}}{360} \right)^{-1}$$

Now, effective return

$$\begin{aligned} \text{EAY}(\mathbf{i}_e) &= (1 + \text{HPY})^{365/\mathbf{N}_{\text{SM}}} - 1 = \left(1 + \frac{\mathbf{P}_1 - \mathbf{P}_0}{\mathbf{P}_0} \right)^{365/\mathbf{N}_{\text{SM}}} - 1 \\ &= \left(\frac{\mathbf{P}_1}{\mathbf{P}_0} \right)^{365/\mathbf{N}_{\text{SM}}} - 1 = \left[\left(1 - \frac{\mathbf{dN}_{\text{SM}}}{360} \right)^{-1} \right]^{365/\mathbf{N}_{\text{SM}}} - 1 \end{aligned}$$



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- If we know HPY, then:
- $EAY = (1 + HPY)^{365/t} - 1$
- $r_{MM} = HPY \times 360/t$
- If we know EAY, then:
- $HPY = (1 + EAY)^{t/365} - 1$
- $r_{MM} = [(1 + EAY)^{t/365} - 1] \times (360/t)$
- If we know r_{MM} , then:
- $HPY = r_{MM} \times (t/360)$
- $EAY = (1 + r_{MM} \times t/360)^{365/t} - 1$

INTEREST RATE FUTURES



WHAT ARE IRFs

- **An interest rate future is a financial derivative with an interest-bearing instrument as the underlying asset. It is a particular type of interest rate derivative.**



COMMON INTEREST RATE FUTURES

- **T BILL FUTURES**
- **EURODOLLAR FUTURES**
- **T-BOND FUTURES**



US T-BILLS

- A short-term debt obligation backed by the U.S. government.
- T-bills are sold in denominations of \$1,000 up to a maximum purchase of \$5,000,000.
- Commonly have maturities of one month (4 weeks), three months (13 weeks), six months (26 weeks) and 1 year.



CONTRACT SPECIFICATIONS FOR US T-BILL FUTURES

Underlying Asset	Usually 3 month T-Bill at the time of issue sold at a discount and quoted at Actual/360 day basis
Unit of trading	USD 1.00 million (FV of T-bill covered by one futures)
Delivery Months	Mar/June/Sept/Dec
Mode of Quote	IMM Index ($100 - y_f$)
Tick Size	0.50 BP = USD 12.50
Settlement	90,91,92 day T-Bill depending on the months in the contract, physical delivery.



- **What is delivered is a T-bill with 3 months to maturity and a million dollar face value.**
- **Three months can mean 90, 91, or 92 days, depending on the month in which the contract was issued.**
- **These bills are auctioned at regular intervals, and the delivery dates of the futures contracts are set so that on a delivery date, there are three possible underlying T-bills which can be used to meet the delivery obligation: the new 3 month T-bill, the "seasoned" 6 month T-bill, and the "seasoned" 1 year T-bill.**



- **The seasoned 6 month bill is that which was issued three months ago and has three months left to maturity.**
- **The seasoned one year bill was issued nine months ago and also has three months left.**
- **Settlement occurs only in March, June, September, and December.**

- These dates set up a cycle of traded contracts.
- For example, in March, three T-bill futures contracts that are trading are those for delivery in:
- June (of a currently 6 month T-bill),
- September (of a T-bill which currently has nine months to maturity),
- December (of the T-bill with one year to maturity).
- With each of these contracts, the underlying bond is also traded.



- **However, you can trade futures with delivery dates even up to two years in the future.**
- **For the longer maturity dates, the underlying bond does not trade while the futures contract does.**

SUMMARY

- On a given delivery date, three T bills are available for settlement:
- the new 3 month bill,
- the 6 month bill issued 3 months ago,
- the 12 month bill issued 9 months ago.
- On a given trade date (e.g. March), three futures contracts are available for trading:
- June delivery of the bill maturing in Sept,
- Sept delivery of December maturing bill and
- Dec delivery of March maturing bill.

QUOTED OR “INDEX” PRICE & “TRADED” PRICE

- The IMM “quoted” or “Index” price $Q_f = 100 - y_f$ where y_f is the quoted discount yield on the futures contract.
- The actual futures traded value at which the contract is traded per F of nominal value is

$$P_{\text{trade}} = F \left(1 - \frac{y_f}{100} \frac{90}{360} \right)$$



- If the contract face value is z , then traded value per contract (contract value) is

- $$P_{\text{trade}} = z \left(1 - \frac{y_f}{100} \frac{90}{360} \right)$$

- NSE: $z = \text{INR } 200,000$

- CME: $z = \$1,000,000$

- **QUOTATIONS BASED ON YIELDS AND NOT ON INTEREST RATES. y_f IS THE FUTURES DISCOUNT YIELD.**
- **QUOTATIONS BASED ON 90 DAYS INSTRUMENT, ACTUALLY SETTLEMENT DELIVERY MAY BE SLIGHTLY DIFFERENT.**



- The contract is designed so that a one-basis-point ($=0.01\%$) move in the futures quote corresponds to a gain or loss of \$25 per contract.
- When a T-Bill futures quote increases by one basis point, a trader who is long one contract gains \$25 and a trader who is short one contract loses \$25.

- **Discount Yield vs Interest Rate**

$$P = F(1 - yT)$$

$$F = P(1 + rT)$$

$$P = F(1 + rT)^{-1} \simeq F(1 - rT)$$